

(2) A guitar string tuned to G (392 Hz) sags in the middle, owing to its own weight, by what distance?

If you recall that a weight hung on a massless spring oscillates with the frequency of a pendulum of length equal to the extension of the spring by the weight, you will not be surprised that the length and mass of the guitar string were not given, and you will be able to predict with some confidence the order of magnitude of the sag distance  $s$ . It should be  $s \approx g/4\pi^2 f^2$ , the length of a pendulum of frequency  $f$ . An accurate calculation is easy, too. For so shallow a catenary an excellent approximation is the parabola  $y = 4s x^2/L^2$ , where  $L$  is the length of the string. If  $T$  is the tension in the string and  $\rho$  its mass per unit length, the string's weight  $L\rho g$  is supported by the upward force component  $T dy/dx$  at each end of the horizontal string. But  $T = 4L^2 \rho f^2$ , and putting all this together, we find that  $s = g/32f^2$ . With  $f = 392$  Hz and  $g = 980 \text{ cm s}^{-2}$ , the sag  $s$  is  $2.0 \mu\text{m}$ . Of course, guitar strings aren't necessarily horizontal. If that bothers you, make it a grand piano.