

(3) A perfect heat engine, or combination of heat engines, has as its only input equal amounts of hot water at  $90^\circ\text{C}$  and cold water at  $10^\circ\text{C}$ . As its only output it squirts all this water out in one high-speed jet. What can you say about the temperature and speed of the water in the jet?

Assume the specific heat of water,  $c_v$ , is constant between the temperature of the cold water,  $T_1 = 283\text{ K}$ , and that of the hot water,  $T_2 = 363\text{ K}$ . If the process does not change the total entropy the temperature  $T$  of the water in the jet must be such that  $\ln(T/T_1) - \ln(T_2/T) = 0$ . Hence  $T = (T_1 T_2)^{1/2} = 320.5\text{ K}$ , or  $47.5^\circ\text{C}$ . The thermal energy of a kilogram of cold water has been increased by  $c_v(T - T_1)$  while that of a kilogram of hot water has been lowered by  $c_v(T_2 - T)$ . The difference, in this case  $5.0 c_v$ , becomes the kinetic energy of 2 kg of water in the jet. Taking  $c_v = 4000\text{ J/kg deg}$ , we get  $140\text{ m s}^{-1}$  for the speed of the jet. These are limiting values. Any irreversibility in the process will result in a higher temperature and lower velocity for the output water. In fact, to achieve the ideal performance we would need a sequence of heat engines, operating over infinitesimal temperature steps, to transfer heat reversibly from the hot to the cold water.