

(1) How fast can a 10-milligram water drop spin without flying apart?

Consider an isolated spherical drop of radius a spinning freely at angular speed ω_0 . Its rotational inertia I_0 is $(8\pi/15)\rho a^5$; its kinetic energy is $(1/2)I_0\omega_0^2$. Suppose the drop while spinning freely deforms to a different shape with $I > I_0$. Its angular momentum remains constant while its kinetic energy is reduced, making energy available to increase the surface area of the drop from $4\pi a^2$ to A . If S is the surface tension, about 50 erg cm^{-2} for water, the resulting increase in surface energy is $(A - 4\pi a^2)S$. Now imagine a spinning sphere which becomes two spheres of smaller radius orbiting around an axis to which they are tangent. This object is clearly on the brink of fission. The surface area has increased by the factor $2^{1/3}$. The rotational inertia has increased by the factor $7/2^{5/3}$, or 2.205. If the decrease in kinetic energy equals the increase in surface energy the initial angular velocity ω_0 must satisfy the relation $\omega_0^2 = 21.5 S/M$, where M is the mass of the drop. The example shows us that a modest increase in surface area (here only $2^{1/3} - 1$, or 26 percent) can permit a drastic change in shape. (For any small deformation of the sphere, of course, the increase is of second order.) For the 10-milligram drop the relation above gives $\omega_0 = 330 \text{ s}^{-1}$, about 50 revolutions per second. Without knowing something about the relative stability against various modes of deformation, and using much more than one envelope, I can't make a better guess.