

(1) Which is larger in absolute magnitude, the thermal energy of matter in the Earth or the (negative) gravitational self-energy of the Earth?

The work done in dispersing to infinity all Earth material must be given by $CMgR$, where M is the Earth's mass, R is its radius, g is the acceleration of gravity at the surface, and C is a dimensionless factor of order unity. The value of C depends on how the mass is distributed. Its value is $\frac{3}{5}$ for a sphere of uniform density, as a simple integration would show, and will be somewhat larger than that if the density is higher near the center, as is the case in the Earth. Let's provisionally set $C = 1$. Then, with $M = 6 \times 10^{24}$ kg, $g = 10 \text{ m s}^{-2}$, and $R = 6 \times 10^6$ m, the negative gravitational energy is 3.6×10^{32} J. The thermal energy depends on the specific heat and the mean temperature. For the former, we will treat the Earth as a solid made of $M \times 6 \times 10^{27} / A$ atoms of atomic weight A , with the specific heat per atom of a three-dimensional harmonic oscillator, 4×10^{-23} J/deg. The mean atomic weight is surely less than 50 and more than 20, for the Earth is mostly silicon, oxygen, magnesium, and iron. If we assume $A = 30$, we cannot be off by as much as a factor of 2 either way. This gives us 860 J/deg for the mean specific heat per kilogram. We know the interior of the Earth is hot—at least as hot as the glowing lava that erupts from it. A few thousand degrees seems a reasonable guess for a mean interior temperature. To be definite, we will try 3000 K, obtaining thus for the thermal energy 1.5×10^{31} J. Our estimate of the gravitational energy was more than 20 times greater. It seems unlikely that our approximations and guesses have distorted the comparison by so large a factor. We conclude, though not with overwhelming confidence, that the gravitational energy exceeds the thermal energy by something like a factor of 10.