(2) A ribbon 2 cm wide is drawn at a constant speed of 10 cm/s into a vat of oil and then vertically out of it. The density of the oil is 0.8 gm cm^{-3} and its viscosity is 0.15 poise (dyn cm^{-2} s), or 0.015 Pa s (newton m^{-2} s). Estimate the rate, in cm^3/s, at which oil is being carried away by the ribbon.

For an order-of-magnitude estimate a dimensional argument should suffice. If inertial forces are not important—an assumption to be checked later—the forces involved arise from weight of oil and from viscosity. The former is proportional to ρg, ρ being the density, while the latter is proportional to the viscosity η. Hence ρ, g, and η can only enter as $\eta/\rho g$, a quantity with dimensions cm-s. The thickness t of the oil coating carried away by the rising ribbon can only depend on this quantity and the ribbon's velocity v_0, and must therefore be given by some numerical factor times the length $(v_0\eta/\rho g)^{1/2}$. For the conditions specified this characteristic length is 0.04 cm. This seems reasonable, in order of magnitude. If that is the thickness of the layer of oil on the ribbon we need only multiply by v_0 and the perimeter of the ribbon, 4 cm, to obtain as an order-of-magnitude estimate of the rate of transfer of oil to the ribbon: 1.6 cm^3/s.

A complete solution of this problem is really more interesting and not difficult, although by the time I had found all my mistakes I had used up much more than one envelope. Let $v(x)$ be the upward velocity of oil, within the film, with x measured horizontally outward from the surface of the ribbon. The relation $d^2v/dx^2 = \rho g/\eta$ is the key to the problem. Where the ribbon emerges from the oil $v(x)$ varies from v_0 down to zero and the thickness of the film is $(2v_0\eta/\rho g)^{1/2}$. The mean velocity over the parabolic velocity profile is $v_0/3$, so the rate of upward transport of oil at that level is $\sqrt{2}/3(v_0\eta/\rho g)$ cm^3/s per cm of ribbon width. Much farther up the rate of transport is, in the steady state, necessarily the same. But here the velocity $v(x)$ decreases from v_0 at the ribbon surface only to 0.866 v_0 at the surface of the film, the thickness of which has now shrunk to its asymptotic value 0.517 $(v_0\eta/\rho g)^{1/2}$.

We must still justify our neglect of inertial forces, that is, accelerations. A Reynolds number for the flow will be something like $v_0 \rho t/\eta$ where t is the initial film thickness. This is of order unity in our case, certainly not large compared to unity. Our order-of-magnitude estimate should not be affected. In the complete solution just described the initial flow might be a little different. The asymptotic behavior of the oil layer on the ribbon involves no acceleration.