(3) Imagine a galaxy which contains 10^{11} stars about like the Sun more or less evenly distributed within a sphere of radius 50 000 light-years. This galaxy collides head on with a similar galaxy toward which it had been moving at a speed of 300 km/s. How many stellar collisions are to be expected?

The Sun's radius R is 8×10^{10} cm. The geometrical cross section for collision of two such spheres is $\pi(2R)^2$, or 8×10^{22} cm^2. We can avoid the integration an exact solution would entail by distributing the stars in a cylinder, rather than a sphere, of radius a bit smaller than 50 000 light-years—say 40 000 light-years, or 4×10^{22} cm. As seen from an approaching star, the 10^{11} target stars are spread over an area of 5×10^{45} cm^2. The probability that it hits one of them is 1.6×10^{-12}. As the galaxies pass through one another the expected number of such encounters will be something like 10^{11} times that, roughly 0.2. But here we used the geometrical cross section. Gravitational attraction of two stars increases their collision cross section. To estimate the importance of that in this case, we may compare the potential energy of two stars in contact, $-GM^2/2R$, with their initial kinetic energy in center-of-mass coordinates, $Mv_0^2/4$. For $M = 2 \times 10^{33}$ g, $v_0 = 3 \times 10^7$ cm/s, the ratio $2MG/Rv_0^2$ has the value 3.3. This suggests that the cross section will be significantly increased. An exact calculation, for which another envelope may be needed, shows that it will be larger than geometrical by just the factor $1 + 2MG/Rv_0^2$, or in this case 4.3. So the expected number of collisions is about 1. The probability that no collision whatever occurs is roughly 0.5. We have, however, forgotten another effect, crudely describable as the attraction of the entire galaxies for one another. Let's go back to the envelope and evaluate again the ratio $2MG/Rv_0^2$, but this time with M the galactic mass and R the galactic radius. The result is 0.5, small enough to give us some confidence that our gross over-simplification of the many-body dynamics has not led us astray by an order of magnitude.